HAND-CLUB INTERACTION: 2. MID-HAND POINT VS. CENTER OF ROTATION

Young-Hoo Kwon
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Abbreviations

COM: center of mass
COR: center of rotation
MH: mid-hand

Introduction

It appears that the current controversy on hand-club interaction moment (torque) stems from selection of the point of interaction used in the analysis. The inverse dynamics group (including Sasho MacKenzie and myself) uses the fixed point perspective while the center of rotation (COR) group (including Steven Nesbit) uses the floating point (i.e. COR) perspective. I use the mid-hand (MH) point as the point of interaction in the computation of the net hand-club interaction force and moment acting on the grip.

The inverse dynamics approach with a fixed point of interaction essentially views the point as a virtual joint. For example, the MH point is considered as a virtual joint that connects the club with the hands. The net hand-club force and moment computed about this point ultimately reflect the linear and angular efforts put into the hand-club interaction by the virtual joint and the muscles around the joint. The COR perspective may be an innovative new method but the inverse dynamics approach is considered a gold standard in studying joint kinetics in biomechanics.

The purpose of this article is two-fold: 1) to understand the mechanics of club-hand interaction using both perspectives, and 2) to assess the meaningfulness of the selected point of interaction. Please note that not much information is publically available on the COR perspective as it is a proprietary concept so my description of the COR perspective may not be the same to what Nesbit group is actually using.

Moment of Force Produced by a Force

Moment of force (torque) produced by an eccentric force (i.e. a force that does not pass through the COR) acting on a rotating object is equal to the force magnitude times the moment arm the force forms about the axis of rotation (Figure 1):

$$M = F \cdot d_{\perp}, \qquad [1]$$

where M & F are magnitudes of the moment and force, respectively, and $d_{\perp}$ is the moment arm. Moment arm is the shortest (or perpendicular) distance from the COR to the line of action of the force. To increase the moment, one can increase either the force or the moment arm.

In 3-D space, the moment vector can be expressed as

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}, \qquad [2]$$

where $\mathbf{r}$ is the relative position vector of the point of action of the force to the COR and '$\times$' is the so-called cross product operator:

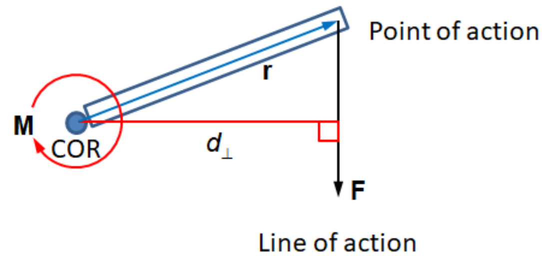

Figure 1. Moment of force produced by an eccentric force

$$\mathbf{M} = \mathbf{r} \times \mathbf{F} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \times \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} yF_z - zF_y \\ zF_x - xF_z \\ xF_y - yF_x \end{bmatrix}. \quad [3]$$

Free Body Diagrams of the Club

Figure 2 shows a free body diagram of the club at a time point during the downswing. $\mathbf{F}_i$ in the diagram is an arbitrary grip force. $\mathbf{r}_i$ is the relative position vector of the point of action of the arbitrary point to the COM of the club. Newton's 2nd Law of Motion gives

$$\sum_i \mathbf{F}_i + \mathbf{W}_C = \dot{\mathbf{p}}_C, \quad [4]$$

$$\sum_i \mathbf{r}_i \times \mathbf{F}_i = \dot{\mathbf{h}}_C, \quad [5]$$

where $\dot{\mathbf{p}}_C \,\&\, \dot{\mathbf{h}}_C$ are the rates of change in linear and angular momentums of the club, respectively, and $\mathbf{W}_C$ is club's weight.

Now in the MH point perspective Eq. 5 can be re-written as

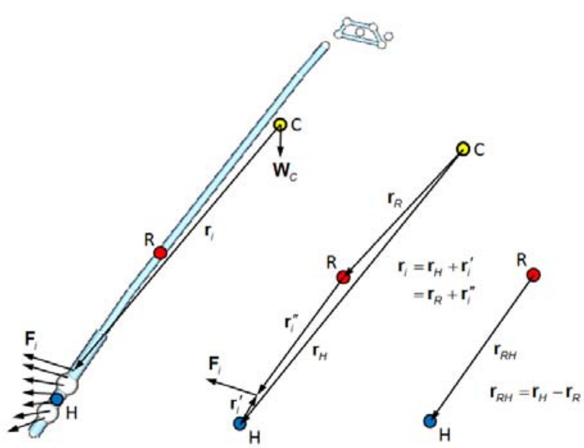

Figure 2. A free body diagram of the club with individual grip forces. Point C is the COM of the club while H and R are the mid-hand point and the instantaneous COR, respectively. Various relative position vectors to club's COM are included here.

$$\dot{\mathbf{h}}_C = \mathbf{r}_H \times \mathbf{F} + \mathbf{M}_H \qquad [6]$$

since

$$\begin{aligned}\sum_i \mathbf{r}_i \times \mathbf{F}_i &= \sum_i \left(\mathbf{r}_H + \mathbf{r}_i'\right) \times \mathbf{F}_i \\ &= \sum_i \mathbf{r}_H \times \mathbf{F}_i + \sum_i \mathbf{r}_i' \times \mathbf{F}_i \\ &= \mathbf{r}_H \times \sum_i \mathbf{F}_i + \sum_i \mathbf{r}_i' \times \mathbf{F}_i \\ &= \mathbf{r}_H \times \mathbf{F} + \mathbf{M}_H .\end{aligned}$$

$\mathbf{r}_H$ & $\mathbf{r}_i'$ (Figure 2) are the relative position of the MH point to the COM and the relative position of the point of action of a grip force ($\mathbf{F}_i$) to the MH point, respectively. $\mathbf{F}$ & $\mathbf{M}_H$ in Eq. 6 are the net MH force and moment of the grip forces about the MH point (Figure 3A), respectively:

$$\mathbf{F} = \sum_i \mathbf{F}_i , \qquad [7]$$

$$\mathbf{M}_H = \sum_i \mathbf{r}_i' \times \mathbf{F}_i . \qquad [8]$$

The net MH moment can be computed from Eq. 6:

$$\mathbf{M}_H = \dot{\mathbf{h}}_C - \mathbf{r}_H \times \mathbf{F}. \qquad [9]$$

Eq. 5 can be run down in the COR perspective as well:

$$\dot{\mathbf{h}}_C = \mathbf{r}_R \times \mathbf{F} + \mathbf{M}_R \qquad [10]$$

since

$$\begin{aligned}\sum_i \mathbf{r}_i \times \mathbf{F}_i &= \sum_i \left(\mathbf{r}_R + \mathbf{r}_i''\right) \times \mathbf{F}_i \\ &= \sum_i \mathbf{r}_R \times \mathbf{F}_i + \sum_i \mathbf{r}_i'' \times \mathbf{F}_i \\ &= \mathbf{r}_R \times \sum_i \mathbf{F}_i + \sum_i \mathbf{r}_i'' \times \mathbf{F}_i \\ &= \mathbf{r}_R \times \mathbf{F} + \mathbf{M}_R .\end{aligned}$$

$\mathbf{r}_R$ & $\mathbf{r}_i''$ (Figure 2) are the relative position of the COR to the COM and the relative position of the point of action of a grip force ($\mathbf{F}_i$) to the COR, respectively. $\mathbf{F}$ & $\mathbf{M}_R$ in Eq. 10 are the net COR force and moment of the grip forces about the COR (Figure 3B), respectively:

$$\mathbf{M}_R = \sum_i \mathbf{r}_i'' \times \mathbf{F}_i . \qquad [11]$$

The net COR moment can be computed from Eq. 10:

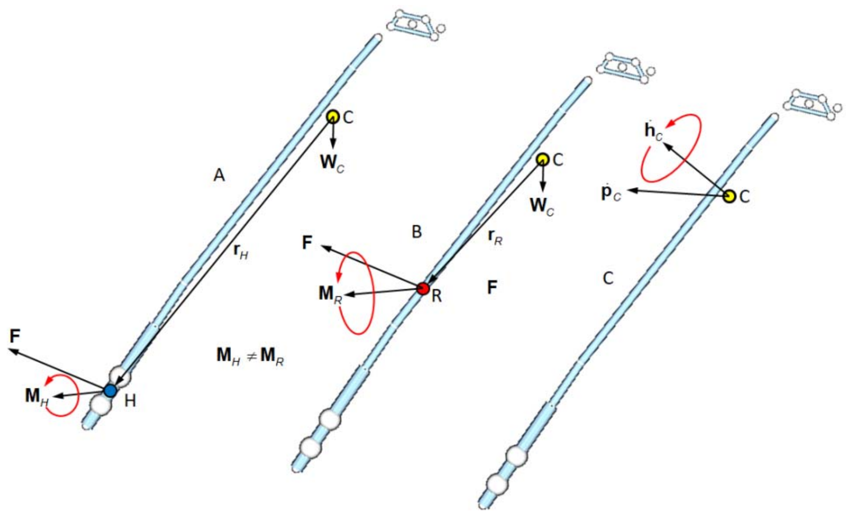

Figure 3. Free body diagrams of the club with net forces and moments computed in different perspectives: the MH perspective (A), the COR perspective (B), and the COM perspective (C). In the first two, the net force remains the same but not the net moment. The COM perspective (C) shows one external force (the inertial force) and one external moment (the inertial moment) each only.

$$\mathbf{M}_R = \dot{\mathbf{h}}_C - \mathbf{r}_R \times \mathbf{F}. \qquad [12]$$

The free body diagram shown in Figure 2 can be replaced with either that of the MH perspective (Figure 3A) or that of the COR perspective (Figure 3B) as all three are mechanically equivalent. Regardless of the perspective used, there are two moments acting on the club: the net moment of the grip forces and the moment generated by the net force about the COM (Eqs. 6 and 10):

$$\mathbf{r}_H \times \mathbf{F} + \mathbf{M}_H = \mathbf{r}_R \times \mathbf{F} + \mathbf{M}_R. \qquad [13]$$

When two different points of interest are used, we will end up with two different sets of net moments and moments generated by the net forces about the COM. The sum of the two, however, should remain the same as it is the inertial moment of the club ($\dot{\mathbf{h}}_C$) as shown in Eq. 13. Figure 3C shows the COM perspective with the inertial force and moment vectors placed at the COM (Newton's 2nd Law of Motion; Eqs. 4 and 5).

Interpretations

The current controversy on assessment of the hand-club interaction moment could stem from the fact that the inverse dynamics approach uses $\mathbf{M}_H$ (Eq. 8) in the assessment while the COR approach uses something else instead. Since not much information is publically available on the hand-club interaction moment used by the COR-based group, I simply assume that their swing (alpha) moment is some sort of measure of hands' effort put on the club such as $\mathbf{M}_R$ (Eq. 11).

In the inverse dynamics approach, the MH point can be considered as a combined virtual joint that links the hands as a group to the club. Since both hands hold the club together, the thorax, the shoulder girdles, and the arms form a closed chain within an open chain and the wrist joints cannot be separated in the inverse dynamics calculations unless an assumption is made. $\mathbf{M}_H$ can be interpreted as the net joint moment produced by the hand/wrist muscles about the combined virtual joint. $\mathbf{M}_H$ thus bears a meaningful information in regards to the efforts put by the hand/wrist muscles in the hand-club interaction.

$\mathbf{M}_R$ in Eq. 11, on the other hand, is the sum of the moments individual grip forces generate about the COR. The problem with this net moment is that the location of the COR changes as the swing progresses. As shown in Figure 4, the COR (green line with red dots) stays in the distal part of the club at event TB. (See the Swing Events & Phases page for detailed definitions of the swing events.) By the time the club gets to the ED position, the COR reaches the distal end of the grip. At the position marked with '*' the COR is at the MH point. At MD, the COR reaches the proximal end of the grip and goes outside thereafter.

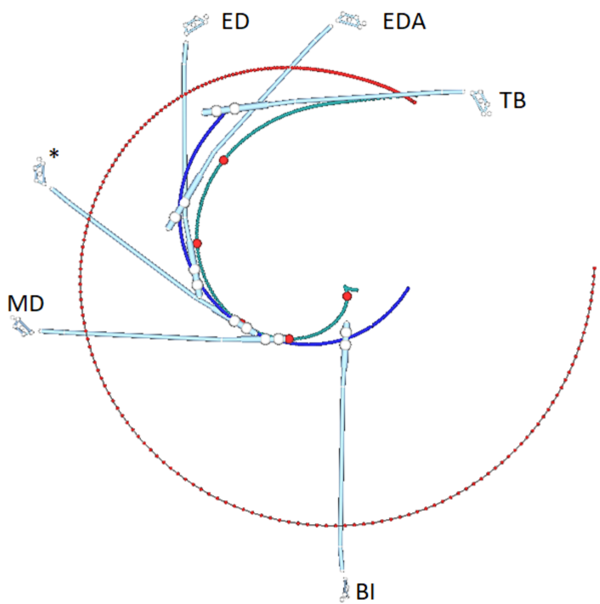

Figure 4. Trajectories of club's instantaneous COR (green line), COM (red line), and MH point (blue line) during downswing. The COR is located close to the clubhead at the beginning of the downswing but moves toward the grip and gets out of the grip near the impact.

From Eqs. 8 and 11, a direct relationship between $\mathbf{M}_R$ and $\mathbf{M}_H$ can be derived:

$$\mathbf{M}_R = \mathbf{r}_{RH} \times \mathbf{F} + \mathbf{M}_H, \qquad [14]$$

since

$$\begin{aligned}\sum_i \mathbf{r}_i'' \times \mathbf{F}_i &= \sum_i \left(\mathbf{r}_{RH} + \mathbf{r}_i'\right) \times \mathbf{F}_i \\ &= \sum_i \mathbf{r}_{RH} \times \mathbf{F}_i + \sum_i \mathbf{r}_i' \times \mathbf{F}_i \\ &= \mathbf{r}_{RH} \times \sum_i \mathbf{F}_i + \mathbf{M}_H \\ &= \mathbf{r}_{RH} \times \mathbf{F} + \mathbf{M}_H.\end{aligned}$$

$\mathbf{r}_{RH}$ in Eq. 14 is the relative position of the MH point to the COR (Figure 2). As evidence by Eq. 14, $\mathbf{M}_R$ is simply sum of the net MH moment ($\mathbf{M}_H$) and the moment generated by the net MH force about the COR ($\mathbf{r}_{RH} \times \mathbf{F}$).

If the alpha moment used by the COR-based group is a measure of the net angular efforts exerted by the hands about the COR, there are two candidates: $\mathbf{r}_{RH} \times \mathbf{F}$ (the moment generated by the net MH force about the COR) and $\mathbf{M}_R$ (total moment generated by individual grip forces about the COR). Figure 5 shows three COR-based swing moments of a PGA Tour-caliber player (driver). The red line is the net MH moment ($\mathbf{M}_H$; Eq. 8) while the green and blue lines are the moment produced by the net MH force about the COR ($\mathbf{r}_{RH} \times \mathbf{F}$) and the net moment of the grip forces about the COR ($\mathbf{M}_R$). The moment produced by the net MH force about the COR (green line) in general is substantially smaller in magnitude than the other two moments throughout the downswing. Therefore, the net moment about the COR (blue line) is fairly similar to that about the MH point (blue line) in terms of magnitude and direction. The net moment about the COR starts positive (counterclockwise) and then turns to negative (clockwise) near event MD.

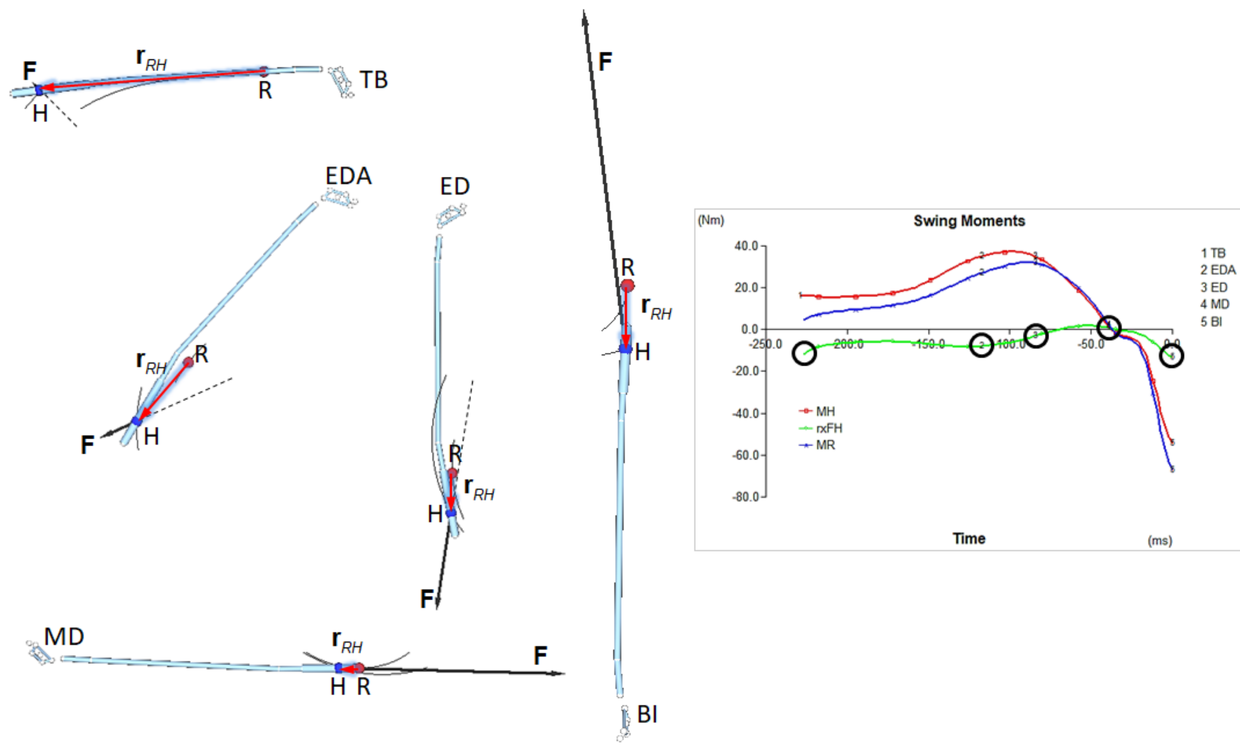

Figure 5. COR-based swing moments: net moment about the MH point ($\mathbf{M}_H$; red line), moment generated by the MH force about the COR ($\mathbf{r}_{RH} \times \mathbf{F}$; green line), and net moment about the COR ($\mathbf{M}_R$; blue line). The stick figures show the net MH force ($\mathbf{F}$) and the relative position of the MH point to the COR ($\mathbf{r}_{RH}$) at various downswing events.

Summary

In this article, two different perspectives of swing moment calculation were explored: the MH-centric inverse dynamics perspective (Figure 3A) and COR-centric perspective (Figure 3B). In the MH-based perspective, the MH point is viewed as the virtual joint that connects the hands collectively to the club. The net MH force and moment thus are the net linear and angular efforts of the virtual joint and the muscles around the virtual joint.

If the swing moments (moments about the axis perpendicular to the swing plane) in question measure the total angular efforts the hands put to the club, it could be either the net moment about the MH point ($\mathbf{M}_H$) or about the COR ($\mathbf{M}_R$). Another candidate in the COR perspective is the moment generated by the net MH force about the COR ($\mathbf{r}_{RH} \times \mathbf{F}$). However, since either the magnitude of the net force is small or the moment arm formed by the net force about the COR is short, the moment generated by the net MH force about the COR is fairly small and the $\mathbf{M}_R$ pattern is quite similar to the $\mathbf{M}_H$ pattern, as shown in Figure 5. Both net moments start positive at event TB and turn to negative near event MD. Since the COR is a floating point that keeps moving relative to the club throughout the swing, comparison of $\mathbf{M}_R$ values between different time points has no meaning. They are based on different COR positions.

This issue can also be assessed in the perspective of practical meaningfulness. To the golfer, the COR position at a given time point is not that obvious as it keeps moving and its location is a function of the shape of club motion. In other words, the golfer cannot necessarily feel the location of the COR during the swing. So how meaningful or useful will the net moment computed about the instantaneous COR ($\mathbf{M}_R$) in golf teaching?

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