HAND-CLUB INTERACTION: 1. INVERSE DYNAMICS

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Abbreviations

- COM: center of mass
- COR: center of rotation
- MH: mid-hand
- MOI: moment of inertia

Introduction

'Inverse dynamics' is a procedure commonly used in biomechanics. Newton's 2nd Law of Linear Motion states "external forces acting on a body accelerate the body. The magnitude of the acceleration ($\mathbf{a}$) generated is proportional to the total (net) force acting on the body and inversely proportional to the mass of the body (m):"

$$\mathbf{a} = \frac{\sum_i \mathbf{F}_i}{m}, \qquad [1]$$

where $\mathbf{F}_i$ is an external force acting on the body. So if the causes of motion (forces) are known, the effect can be predicted ('forward dynamics'). The same relationship can also be presented as

$$\mathbf{F}_{net} = \sum_i \mathbf{F}_i = m\mathbf{a}, \qquad [2]$$

i.e. the cause can be assessed based on the effect observed ('inverse dynamics'). The only problem here is that it is not possible to compute individual forces separately and only the net force ($\mathbf{F}_{net}$) may be assessed through inverse dynamics.

In this article, the 'inverse dynamics' procedure will be applied in the analysis of hand-club interaction.

Net Mid-Hand Force and Moment

Figure 1 shows the forces acting on the grip by golfer's hands. The small arrows in Figure 1A are the grip forces acting on the club by the hands. Let's assume here that we have a grip force measurement system consisting of an array of force sensors covering the entire grip. Then each sensor will pick up the force acting on it and we end up with a large number of small grip forces with varying magnitudes and directions. In reality there is no reliable and validated grip force measurement system available at the moment and it is almost impossible to accurately measure these forces directly. Since it is impossible to measure these forces directly and accurately, this individual force perspective is not that useful.

Conceptually, the hand-club interaction can be simplified substantially by reducing individual grip forces to a net force and a net moment (Figure 1B). Computation of the net force should be straightforward as it is the vector sum of all the forces:

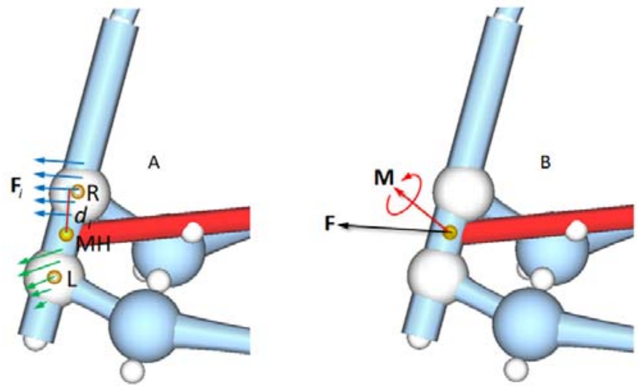

Figure 1. Hand-club interaction: forces acting on the grip by the hands (A) and the net mid-hand force and moment (B)

$$\mathbf{F} = \sum_i \mathbf{F}_i \ . \qquad [3]$$

Computation of the net moment, however, requires selection of a point of interaction as the moment produced by a force must be about a specific point. Let's use the mid-hand point (MH), center of the hands, as the point of interaction for the time being. Each small force ($\mathbf{F}_i$) acting on the grip forms a moment arm (d_i) about the MH point (Figure 1A). 'Moment arm' is the shortest (perpendicular) distance from the center of rotation to the line of action of the force. The net moment ($\mathbf{M}$) is the sum of the moments produced by the grip forces about the point of interaction:

$$\mathbf{M} = \sum_i (F_i \cdot d_i), \qquad [4]$$

where F_i is the magnitude of a given grip force. The net moment is sensitive to the selected point of interaction. If one decides to use the center of the left hand (*L* in Figure 1A) in the triple-pendulum perspective, the net force will remain the same but the net moment will change.

Correct interpretation of the net force and net moment is important here. The net force ($\mathbf{F}$) is the sum of all grip forces while the net moment ($\mathbf{M}$) is the sum of all moments produced by the grip forces 'about the MH point.' The two systems shown in Figure 1 (1A vs. 1B) are thus mechanically equivalent.

Note here that a moment is a vector so a simple arrow is sufficient for visualization. However, a circular arrow (as shown in Figure 1B) was added to the moment vector to clearly differentiate a moment vector from a force vector. This approach was used throughout this document.

Free Body Diagram of the Club

Figure 2 shows a free body diagram of the club in which the club is isolated from golfer's body and the environment and all external forces and external moments acting on the club are marked. Two external forces are acting on the club: the net MH force acting on the grip ($\mathbf{F}_{MH}$) and the weight of the club acting at club's COM ($\mathbf{W}_C$).

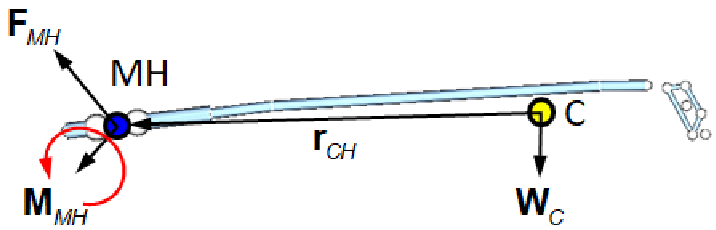

Figure 2. Free body diagram of the club

Eq. 2 gives

$$\mathbf{F}_{MH} + \mathbf{W}_C = \dot{\mathbf{p}}_C , \qquad [5]$$

where $\mathbf{p}_C$ is the linear momentum of the club:

$$\mathbf{p}_C = m_C \mathbf{v}_C , \qquad [6]$$

$$\dot{\mathbf{p}}_C = \frac{d\mathbf{p}_C}{dt} = m_C \mathbf{a}_C . \qquad [7]$$

In Eqs. 6 and 7, m_C is the mass of the club and $\mathbf{v}_C$ & $\mathbf{a}_C$ are the velocity and acceleration of club's COM, respectively. Linear momentum is mass * COM velocity (Eq. 6) and its first time-derivative, the rate of change in linear momentum, is the inertial force: mass * COM acceleration (Eq. 7).

Newton's 2nd Law of Angular Motion states "external moments acting on the club accelerate the club angularly:"

$$\sum_i \mathbf{M}_i = \dot{\mathbf{h}}_C . \qquad [8]$$

where $\mathbf{M}_i$ is an external moment. $\mathbf{h}_C$ in Eq. 8 is the angular momentum of the club about its COM: MOI * angular velocity. In a simple planar rotation,

$$h_C = I_C \omega_C , \qquad [9a]$$

where I_C is club's MOI about its COM and ω_C is club's angular velocity. In a complex 3-D rotation,

$$\mathbf{h}_C = \mathbf{I}_C \boldsymbol{\omega}_C , \qquad [9b]$$

where $\mathbf{I}_C$ is club's inertia tensor (3 x 3 matrix) and $\mathbf{h}_C$ & $\boldsymbol{\omega}_C$ are the angular momentum and angular velocity vectors of the club about its COM, respectively. If the axis of rotation is a principal axis, angular momentum is equal to MOI * angular velocity vector:

$$\mathbf{h}_C = I_C \omega_C, \qquad [9c]$$

and

$$\dot{\mathbf{h}}_C = I_C \dot{\omega}_C = I_C \alpha_C, \qquad [10a]$$

where α_C is the angular acceleration of the club. If the axis of rotation is not a principal axis, the first time-derivative of angular momentum gets a little more complex:

$$\dot{\mathbf{h}}_C = \frac{d(\mathbf{I}_C \omega_C)}{dt} = \frac{d\mathbf{I}_C}{dt}\omega_C + \mathbf{I}_C \frac{d\omega_C}{dt}, \qquad [10b]$$

and is normally presented in different forms such as the Newton-Euler Equation.

For the club system, Eq. 8 can be rewritten as

$$\mathbf{M}_{MH} + \mathbf{r}_{CH} \times \mathbf{F}_{MH} = \dot{\mathbf{h}}_C. \qquad [11]$$

As shown in Eq. 11, two external moments are acting on the club: the MH moment ($\mathbf{M}_{MH}$) and the moment produced by the net MH force ($\mathbf{r}_{CH} \times \mathbf{F}_{MH}$). Since the net MH force does not pass through the COM, it generates a moment about club's COM. $\mathbf{r}_{CH}$ in Eq. 11 is the relative position of the MH point to club's COM (Figure 2). The left-hand side of Eq. 11 is the total (net) moment acting on the club about club's COM and the right-hand side is the inertial moment: MOI * angular acceleration.

Mid-Hand Kinetics

Eqs. 5 and 11 involve two unknowns, $\mathbf{F}_{MH}$ & $\mathbf{M}_{MH}$, so the system can be solved:

$$\mathbf{F}_{MH} = \dot{\mathbf{p}}_C - \mathbf{W}_C, \qquad [12]$$
$$\mathbf{M}_{MH} = \dot{\mathbf{h}}_C - \mathbf{r}_{CH} \times \mathbf{F}_{MH}. \qquad [13]$$

Inertial force ($\dot{\mathbf{p}}_C$), inertial moment ($\dot{\mathbf{h}}_C$), and weight ($\mathbf{W}_C$) of the club and the relative position of the MH point to club's COM ($\mathbf{r}_{CH}$) can be computed through motion analysis. Therefore, the net force (not individual forces) and the net moment (not individual moments) acting on the club by the hands at or about the MH point can be computed from Eqs. 12 and 13 in the inverse dynamics approach.

In the perspective of hand-club interaction, what matters is how much effort is put to the club by golfer's hands both linearly (force) and angularly (moment). The net MH force is the overall linear effort put to the club by the hands while the net MH moment is the overall angular effort put to the club by the hands about the MH point.

Figure 3 shows the on-plane net MH force patterns of a PGA Tour-caliber player during the downswing. Black arrows shown in the stick figure are the net MH force vectors (Eq. 12) at various downswing events. (Visit http://drkwongolf.info/biom/events-phases.html for the definitions of the swing events used.) The green and blue arrows are the tangential and normal axes of the moving MH reference frame.

These two axes are defined on the swing plane. Using this reference frame it is possible to decompose the net MH force into components tangential (green line in the graph) and normal (blue line) to the hand path. It is evident from the force patterns that the net MH force stays outward with respect to the hand path until the position marked with '*' (about 50 ms before the impact) and then turns inward and stays that way thereafter. The tangential component reaches its peak value slightly after event MD where club shaft becomes parallel to the ground. At impact (event BI), the normal force is substantially larger than the tangential force.

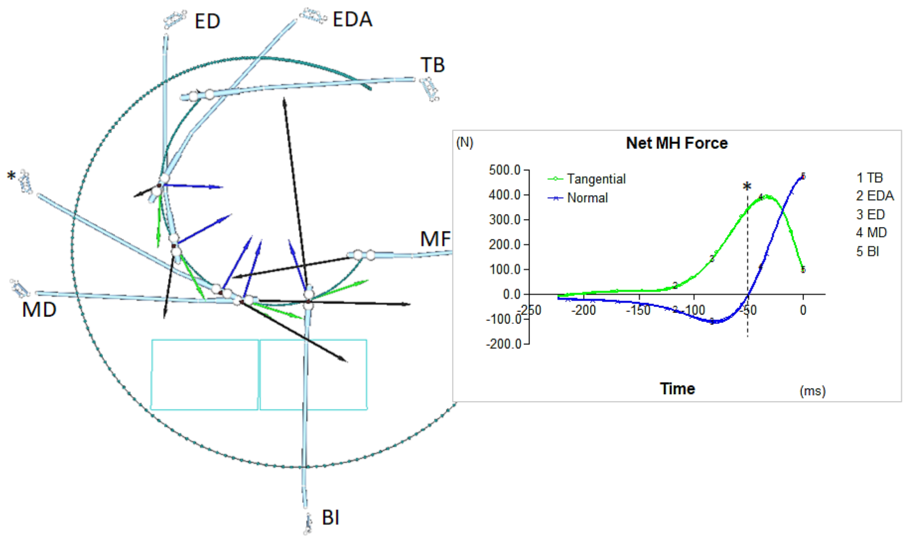

Figure 3. The net MH force (black arrow) acting on the club by the hands. The green and blue arrows are the tangential and normal axes of the moving MH reference frame defined on the swing plane. The net MH force was decomposed to two on-plane components: tangential (green line) and normal (blue line).

Figure 4 shows the swing-axis component of the net MH moment of a PGA Tour-caliber player. Swing axis is the axis perpendicular to the swing plane in the moving MH reference frame. As shown in the figure, this component reaches its positive peak (counterclockwise peak in the swing plane view which promotes rotation of the club in the direction of swing) somewhere between events EDA and ED and its direction changes counterclockwise to clockwise near event MD. A clockwise moment inhibits rotation of the club in the direction of swing. The net MH moment mainly comes from the couple action of the hands about the MH point, and it is an interesting and important observation that the net MH moment generated by the hands about the MH point changes its direction in the middle of downswing near event MD.

Several interpretations have been offered on this. For example, let's imagine a fixed gear bike without 'freewheel.' When the bike moves too fast (perhaps on a downhill) and the pedaling motion of the legs is not fast enough, the leg actions will actually end up inhibiting the rotation of the cranks. Similarly, in the later stage of downswing, if the club rotates (swings) too fast for the wrists to catch up, the hands will end up inhibiting rotation of the club. More importantly, as the clubhead approaches to the impact position, the most important task of the golfer is to generate club motion which secures consistent impacts. When the club rotates (swings) fast at impact, the orientation of the clubface changes rapidly and a small timing error can be translated to a large fluctuation in shot direction. Therefore, slowing

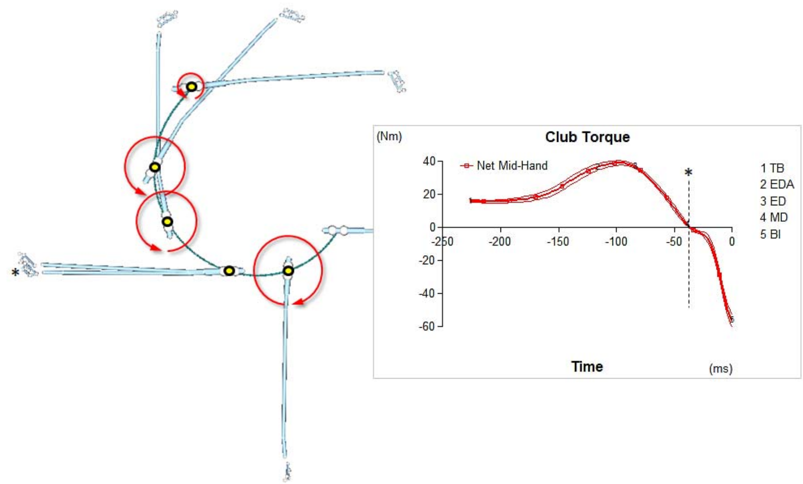

Figure 4. The net MH moment about the swing axis. Direction of the moment changes from counterclockwise to clockwise at the position marked with '*.'

down of the change in clubface orientation (e.g. rate of closure) is very much needed and the net MH moment assumes this role.

In the perspective of the club, the net MH moment is not the only moment acting on it (Eq. 11). Figure 5 shows the swing-axis component of the moment produced by the net MH force about club's COM (let's call this as the 'MH force moment') in the moving MH reference frame. Black broken lines included in the stick figure visualize the lines of action of the net MH forces at various swing events. The swing-axis component of the MH force moment reaches its negative peak (clockwise peak in the swing plane view) between events EDA and ED and its direction changes to positive (counterclockwise) at the position marked with '*' about 50 ms before the impact. This moment can be used effectively to promote down-cocking of the club in early phase of downswing.

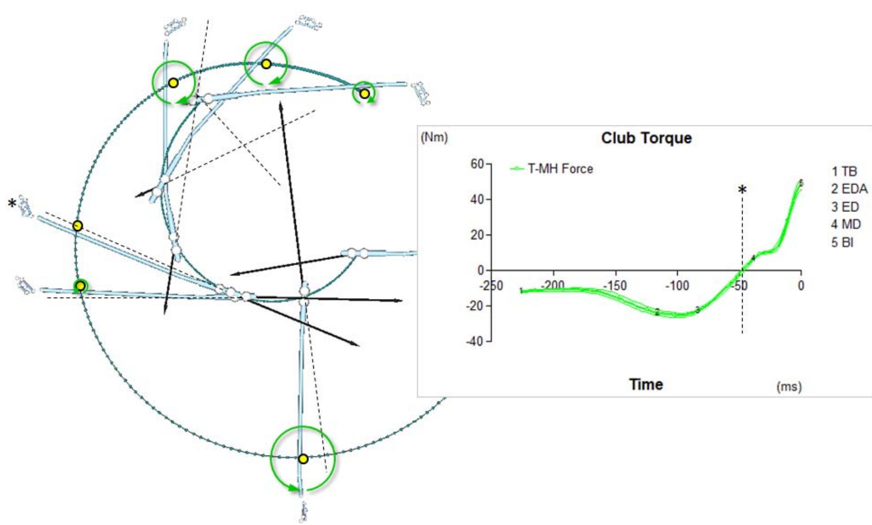

Figure 5. The moment produced by the net MH force about the swing axis. Direction of the moment changes from clockwise to counterclockwise at the position marked with '*.'

Figure 6 provides a summary of the swing-axis moments acting on the club during downswing: the net MH moment (red line), the MH force moment (green line), and the net club moment (blue line). The net MH moment and the MH force moment are in general antagonistic, acting in opposite directions to each other all the time except a short time period before event MD. For most part of the downswing, the net MH moment promotes rotation of the club in the swing direction while the net MH force inhibits rotation of the club. Near the impact, however, their roles switch. Club rotates quite fast near event MD and the couple action of the hands starts interfering with club's rotation here. The net torque which is responsible for angular acceleration of the club largely remains positive (counterclockwise) but turns to negative right before the impact. Slowing down of club's rotation at this stage is beneficial in terms of consistent impacts.

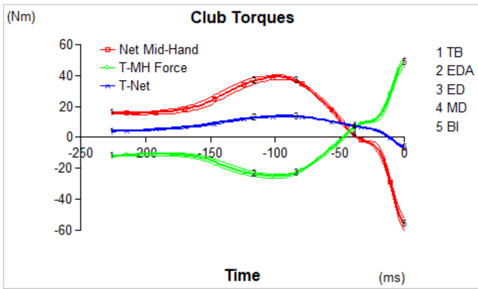

Figure 6. Swing-axis moments acting on the club

The club-centric perspective (Eq. 11; Figures 4-6) definitely allows us to understand how the net MH force and moment contribute to club's angular motion during the swing. One aspect that should be understood clearly here is the fact that only the hands can influence club's motion directly. This is because only the hands are in direct contact with the club.

Selection of the Point of Interaction

The point of interaction is a point that represents the interaction between the hands and club. It was used in defining the net moment acting on the grip by the hands (Eq. 4). The net MH force and moment reflect the net linear and angular effects of the hand-club interactions. In inverse dynamics, the point of interaction can actually be viewed as a virtual joint. Since the thorax, shoulder girdles, and arms form a closed chain within an open chain, it is not possible to separate the net forces and moments exerted by individual hands. So the question is: "Which point is a reasonable choice as a virtual joint that connects the hands to the club?"

One can certainly choose a point other than the MH point as the point of interaction, if necessary. For example, you can choose the center of the lead hand as the point of hand-club interaction. When a different point is chosen, however, the net moment changes while the net force remains the same. So on what ground do we choose the point of interaction, the virtual joint that connects the hands to the club?

Here are two general guidelines:

1. Considering the fact that what we are interested here in is assessment of the net angular interaction (net moment; the couple action mainly) between the hands and club, it is logical to choose a point within the area (or space) of hand-club interaction.

2. It must be a fixed point on the club, not a floating one, so that the net moment can have a consistent meaning. If the relative position of the point of interaction to the club changes, the meaning of the net moment also changes as the net moment is sensitive to the location of the point. If, as the swing progresses, the meaning of the net moment changes, comparison of the net moment values among different time points within the same swing becomes meaningless and irrelevant.

Either hand center (lead or trial) can be a legitimate candidate as either of them suffices both guidelines, although the meaning of the net moment must be interpreted accordingly. In the triple pendulum perspective, the lead hand center or the lead wrist can be a good choice as the point of interaction. When both arms are considered in the double-pendulum perspective, the mid-hand point is a logical choice as the link between the hands and the club.

It seems a group of instructors are promoting use of the instantaneous center of rotation (COR) of the club in the interpretation of the hand-club interaction. Figure 7 shows the trajectory of the instantaneous COR of the club during the downswing. The location of the COR was computed through a simple geometric operation. The bottom line is that the instantaneous COR does not qualify for the point of hand-club interaction in the perspective of both guidelines. The instantaneous COR computed from the motion of the club does not necessarily stay within the area of contact and it is a floating point (moving relative to the club). So the practical meanings of any moments computed about the COR should be interpreted outside the context of hand-club interaction.

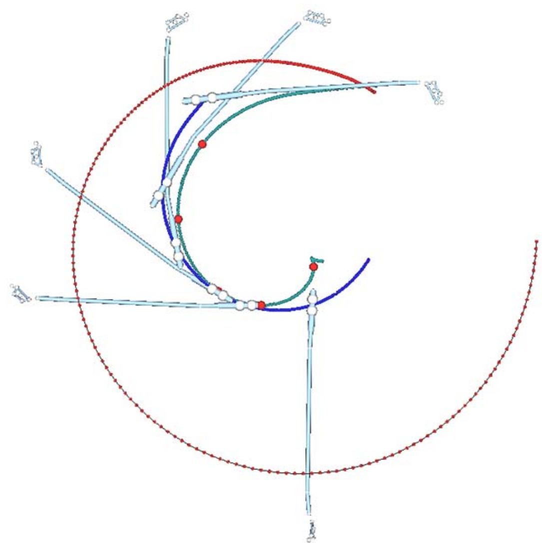

Figure 7. The trajectory of club's instantaneous COR (green line) along with COM (red line) and MH (blue line) paths

Summary

If individual grip forces are available, the hand-club interaction can be assessed directly by the grip forces. With current technology available, however, it is nearly impossible to measure these forces accurately and an alternative approach using the net hand-club interaction force and moment is the only resort. For the computation of the net hand-club interaction moment, selection of the point of interaction (a virtual joint that connects the hands to the club) is necessary and the MH point is a reasonable choice for that. The inverse dynamics procedure coupled with motion analysis allows computation of the net MH force and moment. The net MH force and moment reflect the overall linear and angular effects of the hand-club interactions, respectively.

(Last modified in Feb 2018)